CHAPTER 2

NEUTRON TRANSPORT THEORY

by M. R. Omar

In a reactor core, neutrons move in complicated trajectories due to constant collisions with nuclei. Typically, these recurring collisions cause the neutron trajectories to appear to be zigzag. For instance, source neutrons were originated from their corresponding birth locations, $\mathbf{r}$, moving with particular energy, E, and direction $\widehat{\mathbf{\Omega}}$. Afterwards, they appear at other positions, $\mathbf{r}'$, at a later time. These neutrons could also change its energy and direction into E' and $\widehat{\mathbf{\Omega}}'$, respectively after a collision at $\mathbf{r}'$. In that sense, these neutrons are said to have been transported from the current state $(\mathbf{r}, E, \widehat{\mathbf{\Omega}})$ to the next subsequent state $(\mathbf{r}', E', \widehat{\mathbf{\Omega}}')$. Correspondingly, the study of such a process is coined as the neutron transport theory. In this chapter, an exact equation which describes the neutron transport phenomena will be introduced. Such an equation is recognized as *the neutron transport equation* and the key objective of this study is to solve the equation. The readers will also be introduced with the basic concepts of the neutron transport theory before jumping into the battle of solving the equation.

2.1 Neutron Density and Flux

The central objective of this section is to familiarize the ways of counting the number of neutrons within a nuclear system. In this study, it is essential to understand the approach of characterizing neutrons within a medium. To begin with, we define the *neutron density*, $N(\mathbf{r}, t)\, d^3r$, at a point $\mathbf{r} \in \mathbb{R}^3$ within a reactor core and at time t, as the expected number of neutrons in the unit volume d^3r. It is convenient to characterize neutrons according to their energy, E, and direction, $\widehat{\mathbf{\Omega}}$, such that,

$$
\begin{aligned}
\widehat{\mathbf{\Omega}} = \frac{\mathbf{v}}{|\mathbf{v}|} &= u\mathbf{e}_x + v\mathbf{e}_y + w\mathbf{e}_z \\
&= \underbrace{\sin\theta\cos\theta}_{u}\,\mathbf{e}_x + \underbrace{\sin\theta\sin\varphi}_{v}\,\mathbf{e}_y + \underbrace{\cos\theta}_{w}\,\mathbf{e}_z
\end{aligned}
\tag{2.1}
$$

where $\mathbf{v}$ is the neutron velocity, $\{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\}$ are the basis vectors of the Cartesian coordinate, $\theta \in [0, \pi)$ and $\varphi \in [0,2\pi)$. Correspondingly, the zenith angle, θ, and the azimuthal angle, φ, are indicated in Fig. 2.1. Also,

$$\sqrt{u^2 + v^2 + w^2} = 1 \tag{2.2}$$

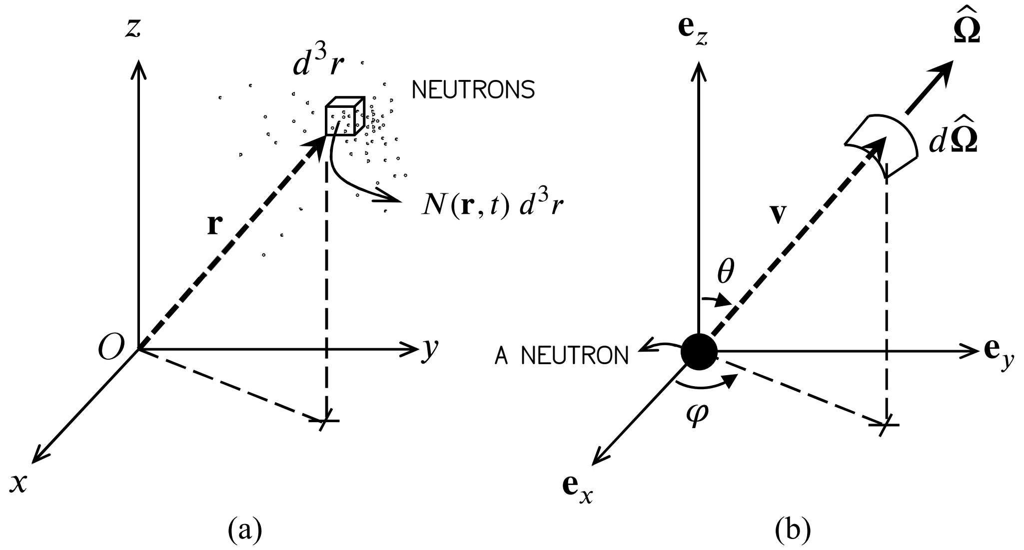

Figure 2.1: (a) The neutron density, $N(\mathbf{r}, t)$; and (b) the direction variables characterizing a neutron.

By referring to Fig. 2.1(a), consider a unit volume d^3r containing a 'mixture' of neutrons with assorted energies and directions. One could possibly select the neutrons with a specific energy E and direction $\hat{\mathbf{\Omega}}$ from d^3r. Thus, the expected number of neutrons in $d^3\mathbf{r}$ at position $\mathbf{r}$, with energy E about dE, moving towards the direction $\hat{\mathbf{\Omega}}$ in solid angle $d\hat{\mathbf{\Omega}}$ at time t is given by,

$$n(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)\ d^3r\ dE\ d\hat{\mathbf{\Omega}} \tag{2.3}$$

Note that the *angular neutron density*, $n(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)$, is introduced in Eq. (2.3). Subsequently, $n(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)$ is defined similarly as $N(\mathbf{r}, t)$, however, the former considers the neutron energy and direction. In the neutron transport theory, it is convenient to express Eq. (2.3) in terms of the *angular neutron flux*, $\psi(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)$, where,

$$\psi(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t) = v\ n(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t) \tag{2.4}$$

where v is the neutron speed ($v = \sqrt{2E/m_n}$, m_n is the neutron rest mass). The angular neutron flux has a unit of cm^{-2} s^{-1}. Subsequently, the *angular neutron current density* is defined as,

$$\mathbf{j}(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t) = \hat{\mathbf{\Omega}}\ \psi(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t) \tag{2.5}$$

Remark that $\mathbf{j}$ is a vector quantity such that,

$$\mathbf{j}(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t) \cdot d\mathbf{A}\ dE\ d\hat{\mathbf{\Omega}} \tag{2.6}$$

is defined as the net rate at which neutrons with energy E about dE and direction $\hat{\mathbf{\Omega}}$ in $d\hat{\mathbf{\Omega}}$ crossing a unit area dA at time t. In certain circumstances, it is favourable to express Eqs. (2.4) and (2.5) without considering the neutron direction, thus,

$$\phi(\mathbf{r}, E, t) = \int_{4\pi} \psi(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)\, d\hat{\mathbf{\Omega}} \tag{2.7}$$

and,

$$\mathbf{J}(\mathbf{r}, E, t) = \int_{4\pi} \mathbf{j}(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)\, d\hat{\mathbf{\Omega}} \tag{2.8}$$

where the integration is taken over the entire solid angle, $d\hat{\mathbf{\Omega}}$, i.e. $[0, 4\pi)$. At this time, the quantity $\mathbf{J}$ is now defined as the *neutron current density*, where,

$$\mathbf{J}(\mathbf{r}, E, t) \cdot d\mathbf{A} \tag{2.9}$$

is the net rate at which neutrons with energy E pass through a surface area $d\mathbf{A}$. Note that the units of both $\mathbf{J}$ and ϕ are equivalent i.e. cm^{-2} sec^{-1}. However, $\mathbf{J}$ is a vector quantity characterizing the *net* rate at which neutron flow through a surface, $d\mathbf{A}$, oriented in a given direction, $d\mathbf{A}/|d\mathbf{A}|$. Whereas ϕ characterizes the total rate at which neutrons pass through an area, regardless of its orientation. Thus, $\mathbf{J}$ is a more convenient quantity to describe neutron leakage from a system such as a nuclear reactor core.

2.2 Neutron Cross Sections

In the neutron transport theory, the concept of neutron cross section is one of the central aspects that determine neutron behaviour within a system. It conveys the likelihood of an interaction between an incident neutron and a target nucleus to occur (Lamarsh & Baratta, 1955). Intentionally, consider a stream of incident neutrons travels through the material within a nuclear reactor. Intuitively, there is a probability that a fraction of these neutrons interacts with the nuclei of the material. The physical quantity that expresses the likelihood of a neutron-nuclear interaction is known as the *microscopic neutron cross section*, $\sigma(\mathbf{r}, E)$. Here, $\sigma(\mathbf{r}, E)$ is a function of the neutron position, $\mathbf{r}$, and the incident neutron energy, E. The former indicates that the σ is dependent on the material properties. Respectively, its value varies across the distinct parts of the reactor core region. Whereas, the latter indicates that the value of σ also varies according to the incident neutron energy. Moreover, the larger the value of σ, the greater the possibility of a neutron-nucleus interaction characterized by σ to occur. Also, σ has a unit of cm^{-2} or "barn" ($\times 10^{-24}$ cm^{-2}).

There is also another form of neutron cross section that considers the likelihood of interaction between an incident neutron and a target nucleus in a chunk of material instead of an individual atom. It is known as the *macroscopic neutron cross section*, $\Sigma(\mathbf{r}, E)$. Formally, $\Sigma(\mathbf{r}, E)$ is defined as the probability of a neutron interaction to happen per unit path length travelled by the neutron. Also, $\Sigma(\mathbf{r}, E)$ has a unit of cm^{-1} and the relationship between $\Sigma(\mathbf{r}, E)$ and $\sigma(\mathbf{r}, E)$ is given by,

$$\Sigma(\mathbf{r}, E) = N_D \sigma(\mathbf{r}, E) \tag{2.10}$$

where N_D is the number of atoms of the material per unit volume or simply called as *the number density*. Equally important, there are various types of neutron-nuclear interactions. These interactions include neutron capture, scattering, fission, and etc. It is now convenient to introduce the microscopic total neutron cross section, $\sigma_{\mathrm{t}}(\mathbf{r}, E)$,

$$\sigma_{\mathrm{t}}(\mathbf{r}, E) = \sum_{j=1}^{m} \sigma_j(\mathbf{r}, E) \tag{2.11}$$

and similarly, the macroscopic total neutron cross section, $\Sigma_{\mathrm{t}}(\mathbf{r}, E)$,

$$\begin{aligned} \Sigma_{\mathrm{t}}(\mathbf{r}, E) &= \sum_{j=1}^{m} \Sigma_j(\mathbf{r}, E) \\ &= N_D \sum_{j=1}^{m} \sigma_j(\mathbf{r}, E) \end{aligned} \tag{2.12}$$

where m is the total number of different types of neutron-nucleus interaction with the summation index, j, representing the different types of interaction.

2.3 Double-differential Scattering Cross Sections

Essentially, $\sigma(\mathbf{r}, E)$ does not conveys the probability of a neutron to possess the outgoing energy, E', and the outgoing direction, $\widehat{\mathbf{\Omega}}'$, after an interaction event. For instance, scattering reaction is a type of interaction that changes the incident neutron energy and direction, i.e. $(E, \widehat{\mathbf{\Omega}})$, into a new set of energy and direction, i.e. $(E', \widehat{\mathbf{\Omega}}')$. Thus, the likelihood of a scattering reaction that causes the change in neutron energy and direction $(E, \widehat{\mathbf{\Omega}})$ into $(E', \widehat{\mathbf{\Omega}}')$ is expressed in term of $\sigma_{\mathrm{s}}(\mathbf{r}, E \to E', \widehat{\mathbf{\Omega}} \to \widehat{\mathbf{\Omega}}')$. At this point, $\sigma_{\mathrm{s}}(\mathbf{r}, E \to E', \widehat{\mathbf{\Omega}} \to \widehat{\mathbf{\Omega}}')$ is known as the *microscopic double-differential scattering cross section*. Now, the dependency of σ_{s} on the incident neutron direction, $\widehat{\mathbf{\Omega}}$, is usually neglected because the nuclei in any material are usually randomly oriented. Consequently, when all possible nuclear orientations are considered, the dependency of σ_{s} on $\widehat{\mathbf{\Omega}}$ averages out.

Crucially, the relationship between $\sigma_{\mathrm{s}}(\mathbf{r}, E)$ and $\sigma_{\mathrm{s}}(\mathbf{r}, E \to E', \widehat{\mathbf{\Omega}} \to \widehat{\mathbf{\Omega}}')$ is given by,

$$\sigma_{\mathrm{s}}(\mathbf{r}, E) = \int_{4\pi} \int_{0}^{\infty} \sigma_{\mathrm{s}}(\mathbf{r}, E \to E', \widehat{\mathbf{\Omega}} \to \widehat{\mathbf{\Omega}}')\, dE'\, d\widehat{\mathbf{\Omega}}' \tag{2.13}$$

Likewise, the property in Eq. (2.10) can also be applied to obtain the macroscopic double-differential scattering cross section,

$$\Sigma_{\mathrm{s}}(\mathbf{r}, E \to E', \widehat{\mathbf{\Omega}} \to \widehat{\mathbf{\Omega}}') = N_D \sigma_{\mathrm{s}}(\mathbf{r}, E \to E', \widehat{\mathbf{\Omega}} \to \widehat{\mathbf{\Omega}}'). \tag{2.14}$$

In addition, both microscopic and macroscopic double-differential cross sections do not depend on the incident neutron direction. However, they depend on the scattering angle, α, which is the angle between $\widehat{\mathbf{\Omega}}$ and $\widehat{\mathbf{\Omega}}'$. It is convenient to express the change in the neutron direction in terms of the scattering cosine, μ_0,

$$\mu_0 = \widehat{\mathbf{\Omega}} \cdot \widehat{\mathbf{\Omega}}' = \cos\alpha \tag{2.15}$$

Finally, one may also express the macroscopic double-differential scattering cross section in terms of μ_0,

$$\Sigma_s(\mathbf{r}, E \rightarrow E', \hat{\mathbf{\Omega}} \rightarrow \hat{\mathbf{\Omega}}') \equiv \Sigma_s(\mathbf{r}, E \rightarrow E', \hat{\mathbf{\Omega}} \cdot \hat{\mathbf{\Omega}}') \equiv \Sigma_s(\mathbf{r}, E \rightarrow E', \mu_0) \tag{2.16}$$

2.4 Neutron Interaction Rate

Ideally, we begin introducing the reaction rate density, R, which is defined as the expected number of neutron-nucleus interactions that occur per unit volume and per unit time. Subsequently, the expected number of interactions per second, f, experienced by a neutron moving with an average speed of υ within the material is given by,

$$f = \upsilon \Sigma(\mathbf{r}, E) \tag{2.17}$$

Thus, the total reaction rate density of a type of interaction in a volume d^3r caused by an incident neutron with energy and direction $(E, \hat{\mathbf{\Omega}})$ at position $\mathbf{r}$ and time t is given by,

$$\begin{aligned} R(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)\, d^3r &= f \times\, n(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)\, d^3r \\ &= \upsilon\, \Sigma(\mathbf{r}, E)\, n(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)\, d^3r \\ &= \Sigma(\mathbf{r}, E)\, \psi(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)\, d^3r \end{aligned} \tag{2.18}$$

The similar concept also relevant to the scattering reaction, where the expected number of scattering reactions that changes the energy and direction of a neutron from $(E, \hat{\mathbf{\Omega}})$ into $(E', \hat{\mathbf{\Omega}}')$ is given by,

$$R(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)\, d^3r = \Sigma_s(\mathbf{r}, E \rightarrow E', \hat{\mathbf{\Omega}} \rightarrow \hat{\mathbf{\Omega}}')\psi(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)\, d^3r \tag{2.19}$$

2.5 Neutron Transport Equation

Principally, the neutron transport theory uncovers the distribution of neutrons in a system. The theory considers the movement of neutrons and the way they interact with the materials contained in the system. The distribution of neutrons in a system, typically in a reactor core, can be obtained by solving the neutron transport equation. One can derive the neutron transport equation by balancing various mechanisms that cause gain or loss of neutrons within a system.

At this instance, it is appropriate to begin deriving the neutron transport equation by considering the rate of change of the neutron density, $n(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)$ within an infinitesimal volume, d^3r,

$$\begin{aligned} \int \frac{\partial}{\partial t} n(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)\, d^3r &= \int \frac{1}{\upsilon}\frac{\partial}{\partial t} \psi(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)\, d^3r \\ &= R_+ - R_- \end{aligned} \tag{2.20}$$

where R_+ is the total rate of interactions that cause the gain of neutrons in d^3r, and R_- is the total rate of interactions that cause the loss of neutrons in d^3r.

2.5.1 Neutron Loss via Net Leakage

At first, consider a few neutron currents, $\hat{\mathbf{\Omega}}\psi$, entering and leaving an infinitesimal volume d^3r of a material through the surface S which defines the boundary of d^3r. In essence, the difference between the rate of neutrons entering and leaving d^3r through S is equal to the resulting neutron leakage rate,

$$\begin{aligned} \begin{Bmatrix} \text{Net} \\ \text{Leakage} \end{Bmatrix} &= \oint_S \mathbf{j}(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t) \cdot d\mathbf{S} \\ &= \oint_S \hat{\mathbf{\Omega}}\, \psi(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t) \cdot d\mathbf{S} \end{aligned} \tag{2.21}$$

Here, Gauss' theorem of vector calculus can be applied, and Eq. (2.21) will reduce into,

$$\begin{aligned} \begin{Bmatrix} \text{Net} \\ \text{Leakage} \end{Bmatrix} &= \int \mathbf{\nabla} \cdot \left(\hat{\mathbf{\Omega}}\psi(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t) \right) d^3r \\ &= \int \hat{\mathbf{\Omega}} \cdot \mathbf{\nabla}\psi(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)\, d^3r \end{aligned} \tag{2.22}$$

2.5.2 Neutron Loss via Disappearance Interactions

Suppose an incident neutron collides with a nucleus in an infinitesimal volume d^3r. Naturally, there is a possibility that an interaction that causes the disappearance of the neutron to occur. If such a disappearance interaction is possible, the neutron is considered loss from d^3r. For example, during a neutron capture interaction e.g. (n, γ) reaction and (n, α) reaction, an incident neutron is absorbed by the nucleus. Consequently, a secondary particle, e.g. γ-ray or α-particle, is released as the product of the reaction. Relevantly, scattering reaction is equally considered as a disappearance interaction. In scattering reaction, an incident neutron with the energy and direction $(E, \hat{\mathbf{\Omega}})$ is considered lost whilst a new secondary neutron with the energy and direction $(E', \hat{\mathbf{\Omega}}')$ is ejected from the nucleus. When a stream of neutrons with energy E and direction $\hat{\mathbf{\Omega}}$ travel through d^3r, the rate of neutron loss in d^3r due to disappearance reactions is given by,

$$\begin{Bmatrix} \text{Total} \\ \text{loss rate} \end{Bmatrix} = \int \Sigma_{\text{t}}(\mathbf{r}, E)\psi(\mathbf{r}, E, \hat{\mathbf{\Omega}}, t)\, d^3r\,. \tag{2.23}$$

Conveniently, all reactions at the collision site are considered as disappearance reactions, thus the total macroscopic neutron cross section is incorporated in Eq. (2.23).

2.5.3 Neutron Gain via In-Scattering

Recall that in the neutron transport theory, neutrons are characterised according to their energy and direction, i.e. $(E, \hat{\mathbf{\Omega}})$. In this section, our objective is to analyse the expected number of neutrons with energy and direction $(E, \hat{\mathbf{\Omega}})$ that appear in an infinitesimal volume, d^3r, due to in-scattering. The term "in-scattering" is coined to indicate the interaction where an incident neutron with any energy and direction, $(E', \hat{\mathbf{\Omega}}')$, are scattered *into* the energy and direction of interest, $(E, \hat{\mathbf{\Omega}})$. For generalization purpose, the neutron in-scattering accounts all types of scattering interaction, e.g. elastic scattering, inelastic scattering, potential scattering, etc.

Most importantly, the gain rate of neutrons with energy and direction $(E, \hat{\mathbf{\Omega}})$ in d^3r due to in-scattering of an incident neutron with energy and direction $(E', \hat{\mathbf{\Omega}}')$ is given by,

$$\left\{\begin{matrix}\text{Total in-}\\ \text{scattering}\\ \text{rate}\end{matrix}\right\} = \int \Sigma_s(\mathbf{r}, E' \to E, \hat{\mathbf{\Omega}}' \to \hat{\mathbf{\Omega}})\, \psi(\mathbf{r}, E', \hat{\mathbf{\Omega}}', t)\, d^3r \tag{2.24}$$

Equally important, to obtain the total neutron gain rate via in-scattering, the sum of the contributions of all incident neutron energies, E', and directions, $\hat{\mathbf{\Omega}}'$, are considered,

$$\left\{\begin{matrix}\text{Total in-}\\ \text{scattering}\\ \text{rate}\end{matrix}\right\} = \int \int_{4\pi} \int_0^\infty \Sigma_s(\mathbf{r}, E' \to E, \hat{\mathbf{\Omega}}' \to \hat{\mathbf{\Omega}})\, \psi(\mathbf{r}, E', \hat{\mathbf{\Omega}}', t)\, dE'\, d\hat{\mathbf{\Omega}}'\, d^3r\,. \tag{2.25}$$

2.5.4 Neutron Gain via Fission

Undeniably, fission reaction is the most important neutron-nucleus interaction that drives the power generation in a nuclear reactor core. A typical nuclear fission reaction such as

$$\mathrm{n} + {}^{235}_{92}U \to {}^{236}_{92}U^* \to \text{fission products}$$

ejects out a mixture of reaction products, including the daughter nuclei and several fission neutrons plus numerous gammas, betas, and neutrinos. The fission fragment nuclei generated by the fission reaction are both highly charged and remarkably high in energy. They slow down through collisions with neighbouring nuclei and dissipating energy during the process. This is, in reality, the primary mechanism by which the fission energy finally appears as heat formed in the fuel material.

Equally important, several neutrons are also produced during the fission reaction. These neutrons can be utilised to breed a fission chain reaction. Majority of these fission neutrons are produced promptly (within 10^{-14} sec) of the fission event and these neutrons are attributed to as prompt (Duderstadt & Hamilton, 1976). Nevertheless, less than 1% of the neutrons produced appear with an apparent time delay from the subsequent decay of radioactive fission products. These delayed neutrons are essential for the practical control of the fission chain reaction. Essentially, the total number of neutrons released in a fission reaction will fluctuate from one reaction to another. However, the average number of neutrons released per fission, v, is of greater concern. This quantity depends on both the nuclear isotope involved and the incident neutron energy. In general, v tends to increase with increasing incident neutron energy (Duderstadt & Hamilton, 1976; Lamarsh & Baratta, 1955).

In particular, fission neutrons are ejected with a distribution of energies, with the average energy being about 2 MeV. Such a distribution will primarily depend on the fissionable isotope involved. The energy distribution may also depend on the incident neutron energy and will vary for prompt and delayed neutrons. It is convenient to introduce the *fission spectrum*, $\chi(E)$, which is defined as the probability of having a fission neutron ejected with energy E as a result of a fission reaction.

Presume that $\nu(E')$ is the average number of neutrons produced per fission induced by an incident neutron with energy E'. Then, the total fission rate at which fission neutrons are generated in an infinitesimal volume d^3r is given by,

$$\int\int_{4\pi}\int_0^\infty \nu(E')\Sigma_f(\mathbf{r},E')\psi(\mathbf{r},E',\hat{\mathbf{\Omega}}',t)\,dE'\,d\hat{\mathbf{\Omega}}'\,d^3r. \tag{2.26}$$

Since we are only interested in knowing rate of fission reaction causing the birth of the fission neutrons with energy E and direction $\hat{\mathbf{\Omega}}$, thus, Eq. (2.26) can be modified into,

$$\chi(E)P(\hat{\mathbf{\Omega}})\int\int_{4\pi}\int_0^\infty \nu(E')\Sigma_f(\mathbf{r},E')\psi(\mathbf{r},E',\hat{\mathbf{\Omega}}',t)\,dE'\,d\hat{\mathbf{\Omega}}'\,d^3r \tag{2.27}$$

where $P(\hat{\mathbf{\Omega}})$ is the probability of having a fission neutron ejected towards the direction $\hat{\mathbf{\Omega}}$. If the fission neutrons are anticipated to get emitted isotropically, then $P(\hat{\mathbf{\Omega}})$ is simply the inverse of all possible solid angles subtended by a fission neutron, i.e. 4π. Finally, the *fission* term of the transport equation is defined as the rate of fission neutron appearing in $(E,\hat{\mathbf{\Omega}})$:

$$\begin{Bmatrix}\text{Fission}\\ \text{rate}\end{Bmatrix} = \frac{\chi(E)}{4\pi}\int\int_{4\pi}\int_0^\infty \nu(E')\Sigma_f(\mathbf{r},E')\psi(\mathbf{r},E',\hat{\mathbf{\Omega}}',t)\,dE'\,d\hat{\mathbf{\Omega}}'\;d^3r. \tag{2.28}$$

2.5.5 The Differential Form of Neutron Transport Equation

At this point, all of the general interaction rate equations that describe the neutron gain and loss mechanisms in d^3r have been expressed. The neutron transport equation can be derived by rewriting Eq. (2.20) in terms of the net rate of neutrons appearing in $(E,\hat{\mathbf{\Omega}})$ and net rate of neutrons loss from $(E,\hat{\mathbf{\Omega}})$,

$$\begin{aligned}\int\frac{1}{v}\frac{\partial}{\partial t}\psi(\mathbf{r},E,\hat{\mathbf{\Omega}},t)\,d^3r &= -\begin{Bmatrix}\text{Net}\\ \text{Leakage}\\ \text{rate}\end{Bmatrix} - \begin{Bmatrix}\text{Total}\\ \text{loss}\\ \text{rate}\end{Bmatrix} + \begin{Bmatrix}\text{Total in-}\\ \text{scattering}\\ \text{rate}\end{Bmatrix}\\ &\quad + \begin{Bmatrix}\text{Fission}\\ \text{rate}\end{Bmatrix}\end{aligned} \tag{2.29}$$

To proceed further, Eqs. (2.22), (2.23), (2.25) and (2.28) are substituted into Eq. (2.29). Finally, the volume integrals over the whole d^3r are cancelled off, and the final form of the neutron transport equation is given by,

$$
\begin{aligned}
&\frac{1}{v}\frac{\partial \psi}{\partial t} + \hat{\boldsymbol{\Omega}} \cdot \boldsymbol{\nabla}\psi(\mathbf{r}, E, \hat{\boldsymbol{\Omega}}, t) + \Sigma_{\mathrm{t}}(\mathbf{r}, E)\psi(\mathbf{r}, E, \hat{\boldsymbol{\Omega}}, t) \\
&\qquad = \int_{4\pi}\int_0^\infty \Sigma_{\mathrm{s}}(\mathbf{r}, E' \to E, \hat{\boldsymbol{\Omega}}' \to \hat{\boldsymbol{\Omega}})\psi(\mathbf{r}, E', \hat{\boldsymbol{\Omega}}', t)\, dE'\, d\hat{\boldsymbol{\Omega}}' \\
&\qquad + \frac{\chi(E)}{4\pi}\int_{4\pi}\int_0^\infty \nu(E')\Sigma_{\mathrm{f}}(\mathbf{r}, E')\psi(\mathbf{r}, E', \hat{\boldsymbol{\Omega}}', t)\, dE'\, d\hat{\boldsymbol{\Omega}}'
\end{aligned}
\tag{2.30}
$$

Alas, solving Eq. (2.30) is rather difficult. It is necessary to simplify the form of Eq. (2.30) before any attempts are made to solve it. One of the well-known simplification methods is via the neutron diffusion approximation. Such an approximation is an essential part in reactor theory since it is adequately uncomplicated to enable detailed calculations. The model is sufficiently realistic to provide many more significant concepts arising in the nuclear reactor analysis. The next chapter will focus on the establishment of an approximate representation of the neutron transport equation, which is much easier to work with.

References

Duderstadt, J. J., & Hamilton, L. J. (1976). Nuclear Reactor Analysis. *Mechanical Engineering*. https://doi.org/10.1109/TNS.1977.4329257

Lamarsh, J. R., & Baratta, A. J. (1955). *Introduction to Nuclear Engineering*. *American Journal of Physics* (Vol. 23). https://doi.org/10.1119/1.1933896