Solution to Assignment questions
JIF 314 Thermodynamics

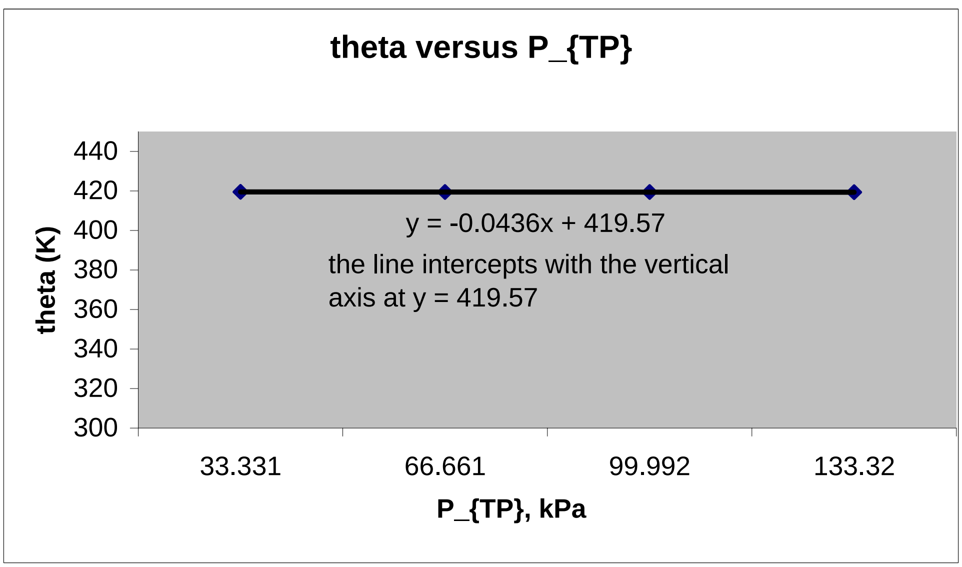
Based on the text book
Heat and thermodynamics by Zemansky and Dittman, 7th edition, Mcgraw-Hill.

Chapter 1

Problem 1.1. Solve using Excel.

First, calculate the value θ of the gas: $\theta = 273.1\text{K}\left(\frac{P}{P_{TP}}\right)$.

P_{TP} (kPa)	P (kPa)	θ (K)
33.331	51.19	419.5211785
66.661	102.37	419.4864944
99.992	153.54	419.4434195
133.32	204.69	419.390342



θ vs. P_{TP} is a straight line in the form of $y = mx + c$, where $y \equiv \theta$, $x \equiv P_{TP}$. The value of θ when P_{TP} becomes zero is the value of the temperature of the gas. This value is simply the value of intersection, c, in the formula of the straight line in the form of $y = mx + c$.

From the formula of the straight line generated by Excell, the intersection of the straight line is $c = 419.57$ in the graph of θ *vs.* P_{TP} .

Hence, the temperature of the gas in the bulb is $\theta = 419.57$ K.

Problem 1.3.

(a) The temperature with resistance measured to be 1000Ω can be calculated using the relationship between R' and T, as per

$$\sqrt{\frac{\log R'}{T}} = a + b\log R', a = -1.16, b = 0.675.$$

Setting R' = 1000 Ω,

$$\left(\sqrt{\frac{\log R'}{T}} = a + b\log R'\right)^2 = \frac{\log R'}{T} = a^2 + b^2(\log R')^2 + 2ab\log R'$$

$$T = \frac{\log R'}{a^2 + b^2(\log R')^2 + 2ab\log R'}$$

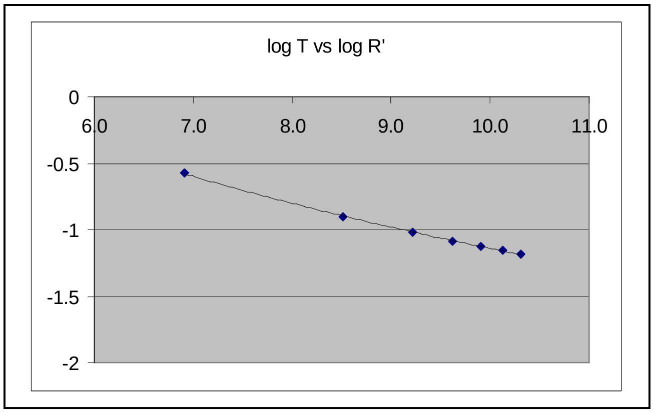
$$= \frac{\log(1000)}{(-1.16)^2 + (0.675)^2\left[\log(1000)\right]^2 + 2(-1.16)(0.675)\log R'}$$

$$= \frac{0.6908}{(-1.16)^2 + (0.675)^2\left[0.6908\right]^2 + 2(-1.16)(0.675)(0.6908)} = 1.44$$

Hence, the temperature of the helium cryostat is 1.44 K.

(b) Use Excell. Plot log R' vs. log T graph by forming the following table:

R’	log R’	T = log R’/(a + b log R’)^2	log T
1000	6.907755	0.563018189	-0.57444
5000	8.517193	0.404427271	-0.90528
10000	9.21034	0.360158153	-1.02121
15000	9.615805	0.338393713	-1.08355
20000	9.903488	0.32444907	-1.12563
25000	10.12663	0.31438398	-1.15714
30000	10.30895	0.306603264	-1.1822



Problem 1.9: $\theta\left({}^\circ\mathrm{F}\right) = \frac{9}{5}\theta\left({}^\circ\mathrm{C}\right) + 32 = \frac{9}{5}(99.974) + 32 = 211.95^\circ\mathrm{F}$ (5 significant figures).

Chapter 2

Problem 2.1

(a) Given the equation of state for a ideal gas $PV = n\,RT$, show that $\beta = \frac{1}{T}$.

Solution:

Given equation of state for a ideal gas

$$PV = n\,RT, \quad \text{Eq. (1)}$$

and the definition of volume expansivity $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)$, it is easily verified that $\beta = 1/T$ by taking the partial derivate of Eq. (1) with respect to T:

$$\frac{\partial}{\partial T}\left(PV = nRT\right) \to P\frac{\partial V}{\partial T} = nR \qquad \text{Eq. (2)}$$

Inserting $PV = nRT$ into Eq. (2), we arrive at

$$\frac{\partial V}{\partial T} = \frac{nR}{P} = \frac{PV}{T}\frac{1}{P} = \frac{V}{T}$$

Hence, $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right) = \beta = \frac{1}{V}\left(\frac{V}{T}\right) = \frac{1}{T}$.

(b) Show that the isothermal compressivility $\kappa = 1/P$.

Solution

Given equation of state for a ideal gas

$$PV = n\,RT, \quad \text{Eq. (1)}$$

and the definition of isothermal compressibility $\kappa = \frac{1}{B} = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)$, it is easily verified that $\beta = 1/P$ by taking the partial derivate of Eq. (1) with respect to P:

$$\frac{\partial}{\partial P}\left(PV = nRT\right) \to P\frac{\partial V}{\partial P} + V = \frac{\partial}{\partial P}\left(nRT\right) = 0 \qquad \text{Eq. (2)}$$

Inserting $PV = nRT$ into Eq. (2), we arrive at

$$\frac{\partial V}{\partial P} = -\frac{V}{P}$$

Hence, $\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right) = -\frac{1}{V}\left(-\frac{V}{P}\right) = \frac{1}{P}$.

Problem 2.2: Given the equation of state of a van der Waals gas, $\left(P + \frac{a}{v^2}\right)(v-b) = RT$, calculate (a) $\left(\frac{\partial P}{\partial v}\right)_T$, (b) $\left(\frac{\partial P}{\partial T}\right)_v$.

Solution:

(a) Taking the partial derivative with respect to v, with constant T,

$$\frac{\partial}{\partial v}\left[\left(P + \frac{a}{v^2}\right)(v-b)\right]_T = \frac{\partial}{\partial v}(RT)\Big|_T = 0$$

$$(v-b)\frac{\partial}{\partial v}\left(P + \frac{a}{v^2}\right)\Big|_T + \left(P + \frac{a}{v^2}\right)\frac{\partial}{\partial v}(v-b)\Big|_T = 0$$

$$(v-b)\left(\frac{\partial P}{\partial v}\Big|_T - \frac{2a}{v^3}\right) + \left(P + \frac{a}{v^2}\right) = 0$$

$$\frac{\partial P}{\partial v}\Big|_T = -\frac{P + \frac{a}{v^2}}{v-b} + \frac{2a}{v^3}$$

(b) Taking the partial derivative with respect to T, with constant v,

$$\frac{\partial}{\partial T}\left[\left(P+\frac{a}{v^2}\right)(v-b)\right]\Bigg|_v = \frac{\partial}{\partial T}(RT)\Bigg|_v$$

$$(v-b)\frac{\partial}{\partial T}\left(P+\frac{a}{v^2}\right)\Bigg|_v + \left(P+\frac{a}{v^2}\right)\frac{\partial}{\partial T}(v-b)\Bigg|_v = R$$

$$(v-b)\left[\frac{\partial P}{\partial T}\Bigg|_v + a\frac{\partial}{\partial T}\left(\frac{1}{v^2}\right)\Bigg|_v\right] + \left(P+\frac{a}{v^2}\right)\frac{\partial v}{\partial T}\Bigg|_v = R$$

$$(v-b)\left(\frac{\partial P}{\partial T}\Bigg|_v + 0\right) + \left(P+\frac{a}{v^2}\right)\cdot 0 = R$$

$$\frac{\partial P}{\partial T}\Bigg|_v = \frac{R}{v-b}$$

(c)

$$\left(\frac{\partial P}{\partial v}\right)_T\left(\frac{\partial v}{\partial T}\right)_P = -\left(\frac{\partial P}{\partial T}\right)_v$$

$$\rightarrow \left(\frac{\partial v}{\partial T}\right)_P = \frac{-\left(\frac{\partial P}{\partial T}\right)_V}{\left(\frac{\partial P}{\partial v}\right)_T} = -\frac{\frac{R}{v-b}}{-\frac{P+\frac{a}{v^2}}{v-b}+\frac{2a}{v^3}} = \frac{R}{P}\left(\frac{1}{1+\frac{2ab}{v^3P}-\frac{a}{v^2P}}\right)$$

Problem 3.2

(a) Show that the work done by an ideal gas during the quasi-static, isothermal expansion from an initial pressure P_i to a final pressure P_f, is given by $W = nRT \ln (P_f/P_i)$.

Solution:

For isothermal process, $P_iV_i = P_fV_f$. Hence $V_i/V_f = P_f/P_i$. Substitute this into $W = -nRT \ln (V_f/V_i)$, we get $W = -nRT \ln (P_i/P_f) = nRT \ln (P_f/P_i)$.

Problem 3.3

An adiabatic chamber with rigid walls consists of two compartments, one containing a gas and the other evacuated; the partition between the two compartments is suddenly removed. Is the work done during an infinitesimal portion of this process (called an adiabatic expansion) equal $P\text{d}V$?

Answer: NO. Because there is no work done against the expansion of the gas-filled compartment by the evacuated compartment.